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On Radiation From A Fiddle-box
As It Is Influenced By the Main Air Resonance
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Note: Surely John Schelleng or someone has worked this out before, but it came through to me about a month ago while teaching my musical acoustics course.

We will confine our attention to the long wavelength limit $kL << 1$ where L is any relevant linear dimension of a fiddle box. In this limit, the only motion of the box walls which can excite the air within it is a net change in box volume during each cycle of oscillation.

(A) Ignoring the motion of the air <u>inside</u> the box, we see that the outside of the box functions as a monopole source of sound of "strength" S_B.

(1) $S_B^{(\omega)}$ = volume flow amplitude associated with the box vibration at frequency ω.

This source will radiate into the room in an isotropic way. It also will drive the box and its f-holes as a Helmholtz resonator.

(B) The driven Helmholtz resonator will contribute a second source of radiation to the outside world. This arises because of the pumping of air into and out of the f-holes. This source has strength $S_H(\omega)$ that is given in terms of S_B as follows:

(2) $$S_H(\omega) = S_B(\omega) \quad \frac{-1}{1 - (\omega/\omega_H)^2 + j(\omega/\omega_H)/Q}$$

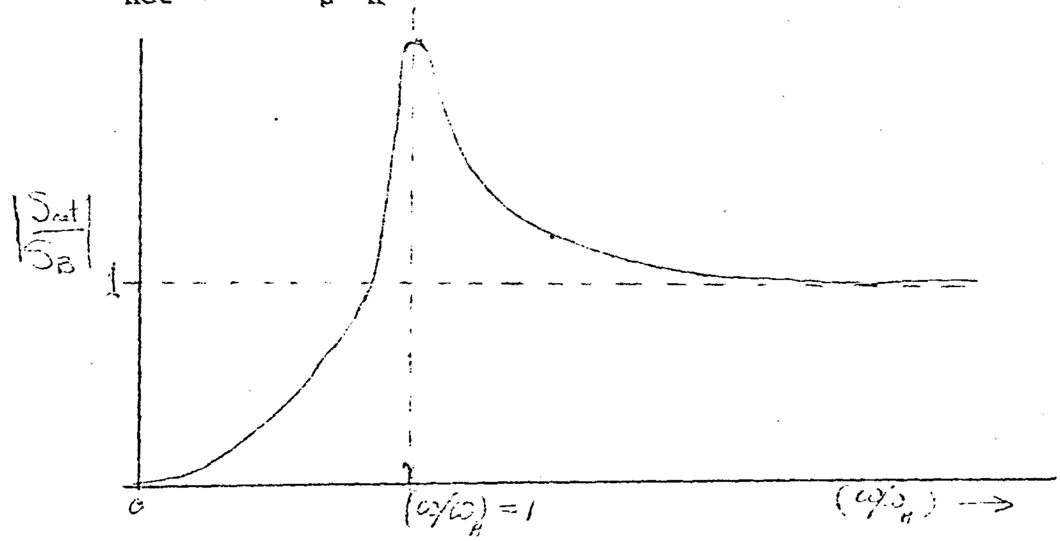
Here ω_H is the natural frequency and Q is the quality factor of the Helmholtz resonator. The minus sign in the numerator arises because the box vibration abstracts air from the interior of the box, (and thus tends to suck air in through the f-holes) during that part of the cycle when its own expansion is pushing air out into the space surrounding the box.

(C) In our long-wavelength approximation it is correct to add the two sources to get the net source strength $S_{net}(\omega)$ of the fiddle box. Well away from resonance, the effects of damping (finite Q) are negligible and we find

(3) $$S_{net}(\omega) = S_B(\omega) \quad \frac{-(\omega/\omega_H)^2}{1 - (\omega/\omega_H)^2}$$

This shows that below about half the Helmholtz frequency, the net source strength is negative and very small (ultimately because the f-holes suck in almost exactly, but slightly more air than the box walls push out, etc., etc.). For frequencies well above ω_H, S_{net} becomes equal to S_B itself. At the resonance frequency, where the influence of damping is most pronounced we find (for $Q \geq 2$)

(4) $$S_{net}(\omega_H) \approx Q S_B(\omega_H)$$



The sketch shows the behavior of the source strength of the box as a function of frequency. The results may be summarized thus:

(D) Below the resonance frequency the fiddle radiates very poorly indeed. One could greatly increase the sound output by stopping up the f-holes. Above the resonance frequency the holes do nothing. At the resonance frequency one gets a Q-fold boost in source strength. The musical implications of this behavior may be drawn when one notices that for low notes (Below about D_4) the higher partials of the tone are radiated strongly relative to the fundamental component whether or not they match the cavity resonance. Above D_4 there is no particular tone-control effect arising from the cavity resonance as it interacts with the wood vibration. The radiation behavior is that normally expected from the surfaces of the box taken by themselves.